

Robustness of the Blandford-Znajek mechanism

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Abstract. The Blandford-Znajek mechanism has long been regarded as a key ingredient in models attempting to explain powerful jets in AGNs, quasars, blazars etc. In such mechanism, energy is extracted from a rotating black hole and dissipated at a load at far distances. In the current work we examine the behaviour of the BZ mechanism with respect to different boundary conditions, revealing the mechanism robustness upon variation of these conditions. Consequently, this work closes a gap in our understanding of this important scenario.

1. Introduction

The Blandford-Znajek (BZ) effect [1] has been proposed as one key mechanism to explain energetic jets from black hole systems. In this mechanism a spinning hole interacts with magnetic fields sourced by an accretion disk (see. e.g. [2, 3, 4]). The magnetic field extracts rotational energy from the spinning black hole and dissipates this energy at a far away load. The details of this process are complex and its understanding requires analyzing the behaviour of the plasma interacting with the strong curvature region around the black holes and how the energy extracted, which is represented by a powerful collimated Poynting flux, is dissipated at large distances.

A simple picture explaining this process is provided by the membrane paradigm [5], where the system is modeled as a circuit composed of a battery – provided by the charge separation induced on the black hole –, two long wires –along magnetic field lines– and a load –at large distances from the black hole–. The energy tapped from the spinning black hole is understood in terms of an induced EMF flowing from the pole to the equator and released at the load. While this picture provides a basic understanding of the underlying phenomena, the role of the load in the resulting jet and energy dissipation has not been the subject of a detailed analysis. In particular, the possibility that different loads could significantly affect the jet has not been explored. If such were the case, this mechanism could be questioned as a robust ingredient to explain black hole systems producing collimated energy outputs.

In this note, in order to investigate this issue, we examine the influence of the boundary conditions on the dynamics of the system and, in particular, on the resulting Poynting flux energy. By considering different conditions, playing the role of the load, we illustrate the robustness of this mechanism. Our results indicate that while these conditions do result in some differences, these do not significantly affect the jet structure and energetics of the system.

2. Physical system

The system is modeled by assuming the force free approximation in a curved background provided by a Kerr black hole spacetime. The black hole is initially embedded in a pure magnetic field configuration, with its intrinsic angular momentum (or spin) aligned with the asymptotic magnetic field. We study the dynamics of the electromagnetic fields numerically and examine the induced Poynting flux. The details of our formulation and implementation are provided in [6, 7]. We here consider more general boundary conditions, which will allow us to examine scenarios corresponding to different resistive loads. To define such conditions we must first understand the characteristic structure of the system

2.1. Characteristic decomposition

Our starting point are the (general relativistic version of the) Maxwell equations. Further, we consider the augmented version of the equations, which include the divergence-cleaning fields $\{\phi, \Psi\}$ to dynamically control the constraint violations [8, 10]. This system is defined by

$$(\partial_t - \mathcal{L}_\beta)E^i - \epsilon^{ijk}\nabla_j(\alpha B_k) + \alpha\gamma^{ij}\nabla_j\Psi = \alpha tr K E^i - \alpha J^i, \quad (1)$$

$$(\partial_t - \mathcal{L}_\beta)\Psi + \alpha\nabla_i E^i = \alpha q - \alpha\kappa\Psi, \quad (2)$$

$$(\partial_t - \mathcal{L}_\beta)B^i + \epsilon^{ijk}\nabla_j(\alpha E_k) + \alpha\gamma^{ij}\nabla_j\phi = \alpha tr K B^i, \quad (3)$$

$$(\partial_t - \mathcal{L}_\beta)\phi + \alpha\nabla_i B^i = -\alpha\kappa\phi. \quad (4)$$

It is illustrative to consider first the electrovacuum case (i.e. $J^i = q = 0$). The characteristic structure can be computed by considering the propagation of perturbations along a generic direction n^i (belonging to an orthonormal tetrad $\{p, q, n\}$, where the index n stands for the longitudinal component). We consider an arbitrary solution (labeled generically by u) and formulate the eigenvalue problem for the perturbed fields (labeled by $[u]$). By defining $\beta^n \equiv \beta^i n_i$ and computing the (Lagrangian) velocities $\tilde{v} \equiv v + \beta^n$, this eigenvalue problem can be written as

$$\tilde{v}[E^i] = -\epsilon^{ijk}n_j[\alpha B_k] + n^i[\alpha\Psi], \quad (5)$$

$$\tilde{v}[B^i] = \epsilon^{ijk}n_j[\alpha E_k] + n^i[\alpha\phi], \quad (6)$$

$$\tilde{v}[\Psi] = \alpha[E^n], \quad (7)$$

$$\tilde{v}[\phi] = \alpha[B^n]. \quad (8)$$

It is now straightforward to obtain the following list of eigenvectors:

- constraint modes: $[\phi] \pm [B^n]$ and $[\Psi] \pm [E^n]$, which involve the divergence-cleaning fields $\{\phi, \Psi\}$ as well as the divergences of E and B . They propagate with light speed $v = -\beta^n \pm \alpha$.
- transversal modes: $[E^i - E^n n^i] \mp [\epsilon^{ijk}n_j B_k]$, which can be written in tetrad components as $[E^p] \pm [B^q]$ and $[E^q] \mp [B^p]$. They correspond to the EM waves and also propagate with light speed $v = -\beta^n \pm \alpha$.

Notice that due to the linearity of the Maxwell equations in vacuum, the eigenvectors depend only on the perturbed fields. Therefore, at this level, our discussion is completely generic and independent of the background solution u .

Let us now consider a more realistic approach to our physical problem, –a spinning black hole interacting with a magnetic dominated, low density plasma, surrounding it–. The effect of this plasma is to provide both a charge q and current distribution J^i which profoundly affect the behavior of the electromagnetic fields. As described in [9, 1], and summarized next, their role is accounted by explicitly defining the component of J^i transversal to B^i and introducing a further constraint to account for the component of J^i parallel to B^i , while q is obtained through Coulomb's law. In the following discussion, we will ignore the contribution of the divergence-cleaning terms since we are interested in describing the physical set-up. In our problem of interest, the plasma inertia is negligible, consequently the Lorentz force vanishes, defining the so-called force-free condition [9, 1],

$$qE^i + \epsilon^{ijk}J_j B_k = 0. \quad (9)$$

This condition determines the current required to close Maxwell equations. The scalar and vector products of the force-free condition (9) with the magnetic field lead respectively to $E_i B^i = 0$ and $J^i = q\epsilon^{ijk}E_j B_k/B^2 + (J_i B^i)B^i/B^2$. The component of the current parallel to the magnetic field can be obtained by imposing in the Maxwell equations $(\partial_t - \mathcal{L}_\beta)(E_i B^i) = 0$, which is a natural consequence of the condition $E_i B^i = 0$. Additionally, in this electrodynamic limit the charge density is defined through the Maxwell constraint $q = \nabla_i E^i$, so that the current only depends on the electromagnetic fields,

$$J^i = (\nabla_m E^m)\epsilon^{ijk}E_j B_k/B^2 + B^i \epsilon^{jkm}(B_j \nabla_k B_m - E_j \nabla_k E_m)/B^2. \quad (10)$$

The force-free evolution system is obtained from the original Maxwell system (1-4), by substituting current (10) in equation (1). Additionally, the field Ψ and its associated evolution equation (2) are eliminated since they become trivial with the definition of $q = \nabla_i E^i$. The eigenvalue problem for the force-free approximation can be written as

$$\begin{aligned}\tilde{v}[E^i] &= -\alpha \epsilon^{ink} [B_k] + \alpha (S^i/B^2)[E_n] + \alpha (B^i/B^2) \epsilon^{jnk} (B_j[B_k] - E_j[E_k]), \\ \tilde{v}[B^i] &= \alpha \epsilon^{ink} [E_k] + \alpha n^i [\phi], \\ \tilde{v}[\phi] &= \alpha [B^n];\end{aligned}\tag{11}$$

where $S^i \equiv \epsilon^{ijk} E_j B_k$ is the Poynting vector. The diagonalization of the system is considerably more involved now due to the non-linearities introduced by the current (10). A convenient rearrangement of the fields makes this task easier, by changing to a basis containing the following combinations

$$\begin{aligned}[C] &\equiv B_q[B_p] - B_p[B_q] + E_p[E_q] - E_q[E_p], \\ [E_t^2 + B_t^2] &\equiv E_p[E_p] + E_q[E_q] + B_p[B_p] + B_q[B_q], \\ [E_i B^i] &\equiv E_n[B_n] + B_n[E_n] + [E_t B^t], \\ [E_t B^t] &\equiv E_p[B_p] + B_p[E_p] + E_q[B_q] + B_q[E_q].\end{aligned}\tag{12}$$

The characteristic problem in this basis is given by

$$\tilde{v} \mathbf{U} = \alpha \begin{pmatrix} S_n/B^2 & B_n(B^2 - E^2)/B^2 & 0 & 0 & 0 & E_n & -1 \\ B_n/B^2 & S_n/B^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -E_n & 0 & 1 & 0 & 0 & 0 \\ -E_n B_n/B^2 & -S_n E_n/B^2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & E_n & 0 & 0 \end{pmatrix} \mathbf{U}\tag{13}$$

with

$$\mathbf{U} = [C \quad E_n \quad S_n \quad E_t^2 + B_t^2 \quad \phi_n \quad B_n \quad E_i B^i]^T.\tag{14}$$

The characteristic matrix can be diagonalized to obtain the following list of eigenvectors

- constraint modes: $[\phi] \pm [B_n]$, which propagate with light speed $v = -\beta^n \pm \alpha$.
- standing mode: $[E_i B^i] - E_n [B_n]$, which propagates with speed $v = -\beta^n$, and contains information on $[E_t B^t]$.
- “Poynting” modes: $-B_n \lambda^\pm [C] + (B^2 \lambda^+ \lambda^- - S_n \lambda^\pm) [E_n] - E_n B_n [B_n] + B_n [E_i B^i]$, which propagate with speed $v = -\beta^n + \alpha \lambda^\pm$, where we have defined $\lambda^\pm \equiv \frac{S_n}{B^2} \pm \frac{B_n}{B^2} \sqrt{B^2 - E^2}$.
- transversal radiative modes: $B^2(\lambda^+ \lambda^- - 1) E_n [E_n] + 2 B_n E_n [E_i B^i] + B^2(\lambda^+ \lambda^- + 1 \mp 2 S_n) ([E_t^2 + B_t^2] \pm [S_n]) \mp 2 E_n B_n [C] \pm 2 E_n^2 B_n [\phi]$, which are a generalization of the standard MHD Alfvén modes, and propagate with light speed $v = -\beta^n \pm \alpha$.

The force-free evolution system is strongly hyperbolic since there is a complete basis of eigenvectors for each direction n^i , ensuring the existence of a unique stable solution within the domain of dependence of the initial hypersurface in a boundary-free case. If the system is to be employed within a finite domain, boundary conditions have to be imposed on (some of) the fields, which may affect the stability of the solution. To ensure the well posedness of the resulting problem care must be taken to define these fields consistently. One way to do so for symmetric hyperbolic systems, relies on

adopting maximally dissipative boundary conditions of the type $U^- = RU^+$, where U^\mp are the ingoing/outgoing eigenvectors of the evolution system, with $|R| < 1$ [11]. In a problem with constraints –as is the case here– further conditions must be derived to ensure the constraints are preserved. In the case of the force-free system as we have shown previously, the characteristic structure is rather complicated and depends on the background solution which complicates the analysis ‡. We defer to a future work the construction of constraint-preserving boundary conditions for the force-free system of equations. Here, since our main interest is to model different behaviours of the resistive load far from the black hole, we will consider the eigenvectors of the electrovacuum Maxwell system as a first step towards consistent and stable boundary conditions for the force-free system. Notice that this is short of defining conditions consistent with a force-free regime, however at far distances from the black hole, the plasma density decreases reducing to the vacuum case. Thus, depending on the physical scenario considered this choice will not be a physical limitation. Nevertheless, regardless of this issue, our approach will allow us to explore the robustness of the BZ mechanism by considering different conditions and assessing the stability and collimation of the resulting (if any) Poynting flux. The boundary conditions considered here take the form

$$([E^i - E^n n^i] - [\epsilon^{ijk} n_j B_k]) = R ([E^i - E^n n^i] + [\epsilon^{ijk} n_j B_k]) , \quad (15)$$

$$([\phi] - [B_n]) = R ([\phi] + [B_n]) , \quad [E_n] = 0 . \quad (16)$$

Notice that there is no rigorous proof that the resulting system will be stable with $|R| < 1$ except in the electrovacuum case. Numerical experiments have shown that it is unstable if $R < 0$ but stable otherwise.

3. Results

We adopt the formulation of the force-free approximation and the numerical implementation already described in [6, 7]. We will consider different boundary conditions corresponding to $R = \{0, 1/2, 1\}$ in order to model the role of the resistive load far away from the source. Notice that we do not claim any of these values to be a true representative of the load –which is unknown in any case–, rather these values allow us to explore the resulting behaviour under profoundly different physical conditions and examine, in particular, the resulting Poynting flux behaviour. Indeed the resulting conditions contemplate that in absence of divergence errors, $[B_n] = 0$, and for:

- $R = 1$: the transversal components of the electric field are free (i.e., determined only by the interior solution) while the transversal components of the magnetic field are set to 0.
- $R = 0$: the transversal components of the magnetic field are equal of the transversal components of the electric field (and determined by the interior solution).

Any other value in $R \in (0, 1)$ just spans the parameter space between these two limits.

To examine the behaviour of the solution under these options we adopt a domain with $(x, y) \in [-12, 12]M$ and $z \in [-16, 16]M$ covered with a uniform grid with grid-spacing given by $\Delta = 0.01M$. We studied the three different cases evolving them

‡ See e.g. [12] for a related example in the MHD case where constraint preserving boundary conditions are defined.

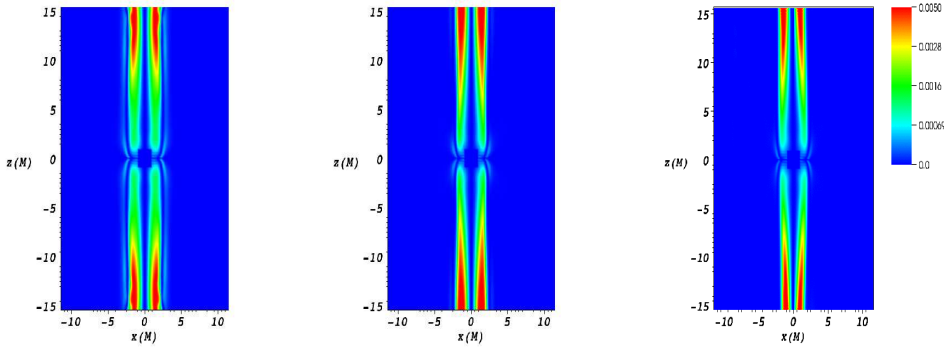


Figure 1. Electromagnetic energy density flux for $R = 0$ (left), $R = 1/2$ (middle) and $R = 1$ (right) at $t = 320M$.

until a quasi-stationary regime is reached, comparing the luminosity obtained for each case far away from the black hole. This luminosity is computed as the integral of the electromagnetic energy flux density (which can be seen for the three cases studied in fig. 1) on a solid angle of 15 degrees along the z -axis (further details on this computation are described in [7]) and shown in figure 2 (left plot), after a convenient renormalization with the asymptotic value of the $R = 1$ case. After a transitory initial behaviour, the obtained luminosities for the different values of the reflection coefficient R are qualitatively the same in structure and the emitted power within similar ranges. It is interesting to notice the energy density flux reaches a rather constant asymptotic configuration in the $R = 0$ case, while relatively small amplitude cylindrical oscillations are observed for $0 < R \leq 1$. Nevertheless, the jet structure and average luminosities obtained are comparable in all cases. Thus, within typical astrophysical uncertainties it is clear the BZ mechanism is not affected when considering significantly different boundary conditions.

To ensure the boundary location does not affect the above observations, we have studied the jet's luminosity for two different boundary placements along the jet's direction; the original domain (small) with $z = [-16, 16]M$ and another one (large) with $z = [-32, 32]M$. Figure 2 (right plot) illustrates the obtained values for the case with $R = 1$ normalized to the asymptotic value of the small domain simulation, showing again that at late times the values obtained are quite similar.

4. Final Comments

This work illustrates the robustness of the BZ mechanism to generate a collimated Poynting flux of energy which is largely independent of the boundary conditions adopted. Consequently, while the load's characteristics are essentially unknown, our studies indicate the jet resulting from the plasma's ability to extract energy from the black hole's vicinity is robust.

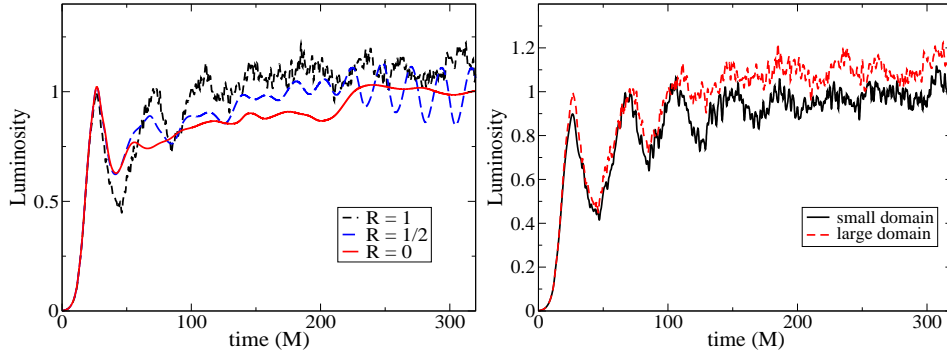


Figure 2. (Left) Luminosity for the three cases considered, normalized to the asymptotic value of the $R = 1$ case. (Right) Luminosity for the two boundary locations considered in the case $R = 1$, normalized to the asymptotic value of the small domain simulation.

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